

**Yaw Angle Tracking Control Design of Underactuated AUV by using State Dependent Riccati Equations (SDRE)-LQT(Desain Kontrol Pelacakan Sudut Yaw AUV yang kurang aktif dengan menggunakan State Dependent Riccati Equations (SDRE) -LQT)**

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**Abstract.** In realizing yaw angle control tracking on AUV, the use of the State Dependent Riccati Equations method based on Linear Quadratic Tracking (SDRE-LQT) is realized. This algorithm calculates changes in yaw angle tracking problems through calculation of parameter changes from online AUV with Algebraic Riccati Equations. So that the control signal given to the plant can follow the changing conditions of the plant itself.

Keywords: AUV; Tracking Control; SDRE-LQT.

**Abstrak.** Dalam mewujudkan pelacakan kontrol sudut yaw pada AUV, penggunaan metode Persamaan Riccati Negara Tergantung berdasarkan Linear Quadratic Tracking (SDRE-LQT) direalisasikan. Algoritma ini menghitung perubahan dalam masalah pelacakan sudut yaw melalui perhitungan perubahan parameter dari AUV online dengan Aljabar Riccati Persamaan. Sehingga sinyal kontrol yang diberikan kepada pabrik dapat mengikuti kondisi perubahan pabrik itu sendiri.

Kata Kunci: AUV; Kontrol Pelacakan; SDRE-LQT.

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I. **INTRODUCTION**

The development of the Autonomous Underwater Vehicle (AUV) has expanded its functions, especially for dangerous underwater military tasks, such as surveillance, search and rescue. Not only for military tasks, AUV can also be used for scientific tasks, namely mapping underwater conditions, detecting oil resources, water maintenance and inspection, and underwater surveys [1, 2]. The main idea in designing a path following control is how to run AUV, so that it moves on a defined path [3–5]. Guidance functions as tracking control of a path in the geometry of space. The tracking control is based on cross track error which is the shortest distance between AUV and path [6]. The State Dependent Riccati Equation (SDRE) control strategy is well known and has become very popular among the control community lately, the use of this algorithm is very effective for addressing nonlinear feedback control problems by considering the nonlinearity state of the system.

In this paper, the AUV model used is a torpedo-shaped model. This model uses five actuators, namely two actuators for sterln, two actuators for rudder and one actuator to function as givers of thrust (Thrust). Although AUV has a simple structure, controlling the motion of AUV is not easy, it is because AUV has nonlinear characteristics, MIMO, uncertainty parameters [3–8]. The uncertainty parameter of the AUV is a parameter in the dynamics of the AUV that changes with time (time varying). The change in dynamics over time is due to the characteristics of the design structure of the AUV, which is determined by the hydrodynamic forces when the AUV passes through a hydro (water) flow [8–10].

Yaw angular direction is the main measure in regulating horizontal motion at AUV, the movement is defined through the steering equation [9]. The equation is used to define 2D horizontal motion. This definition is used to make it easier to calculate what state variables are needed in analyzing horizontal motion and is needed for the need for decreasing steering control laws [9–12]. The reduction in the steering control law is used to design the yaw angle tracking control on the AUV. The tracking control at the yaw angle is used to adjust the direction of the AUV yaw angle to match the given yaw reference signal. The complexity in designing yaw angle tracking control due to the characteristics of the dynamics of AUV is a problem that is not easily solved, so it becomes a challenge for researchers to design it.
From the problems described earlier, we need a method to overcome the problem of nonlinearity and parameter uncertainty using SDRE control, but to be applied to the task in the form of geometry path, AUV requires tracking control in order to minimize cross track errors, the tracking control used is LQT with servo system structure 1.

Thus, this research will propose SDRE-LQT control on the GNC subsystem on AUV.

II. Method of Research

The design of the AUV dynamics is transformed in the form of a matrix so that it can be simulated using Matlab software, the design of the AUV dynamics and the overall design of the control system of the nonlinear model in the form of a matrix:

\[
\dot{\eta} = f(\eta)v
\]

\[
M\ddot{v} + C(v)v + D(v)v + g(\eta) + g_0 = \tau + \tau_{\text{win}} + \tau_{\text{wave}}
\]

The results of the translation of the general equation of motion 6 degrees of freedom above, we get 3 equations for horizontal motion namely surge, sway and yaw.

\textbf{Surge Motion}

\[
X = m[\ddot{u} - vr + wq - x_0(q^2 + r^2) + y_0(pq - \dot{r}) + z_0(pr + \dot{q})]
\]

\textbf{Sway Motion}

\[
Y = m[\ddot{v} - vr + wq - y_0(q^2 + r^2) + z_0(pq - \dot{r}) + x_0(pr + \dot{q})]
\]

\textbf{Yaw Motion}

\[
Z = m[\ddot{w} + wq - x_0(q^2 + r^2) + y_0(pq - \dot{r}) + z_0(pr + \dot{q})]
\]

\textbf{Dynamics of AUV}

At the design stage of AUV dynamics is done by modeling equations (2.1) and (2.2) in the form of a matrix. This modeling aims to determine the dynamics of AUV, so that it can simplify the design of the controller. The first step is to substitute the equation of motion in equations (2.3) to (2.5) with the force equation and external moments in equations (2.6) to (2.8). Furthermore, the components of linear and angular acceleration and linear and angular velocity on the earth axis framework are denoted by \((u, v, w, \phi, \psi, \theta, \phi, \psi, \theta)\) equations (2.3) to (2.5) and (2.6) to (2.8), so that if substituted can be seen in equation (2.9) - (2.11), i.e.

\[
(m - X_0)\ddot{u} - my_0\ddot{r} + mz_0\ddot{q} = -m[ -vr + wq - x_0(q^2 + r^2) + y_0pq + z_0pr + X_{u0}u[u] + X_{uv}uv + X_{ur}ur + X_{w0}w[w] + X_{wq}wq + X_{q0}qq + X_{r0}rr + X_{prop}
\]

\[
(m - Y_0)\ddot{v} - mx_0\ddot{p} + (mx_0 - Y_0)\ddot{r} = -m[ -up + ur - y_0(r^2 + \dot{p}^2) + z_0qr + x_0qp + Y_{HS} - Y_{u0}u[u] + Y_{uv}uv + Y_{ur}ur + Y_{w0}w[w] + Y_{wq}wq + Y_{q0}qq + Y_{r0}rr + Y_{prop}
\]

\[
(l_{xx} - N_r)\ddot{r} - my_0\ddot{u} + (mx_0 - N_0)\ddot{v} = -\{l_{yy} - l_{xx}\}pq + m[x_0(-wp + ur) - y_0(-vr + wq)] + N_{HS} + N_{u0}\delta_s u^2 \delta_s + N_{ur}ur + N_{ux}uv + N_{wp}wp + N_{pq}pq
\]

The kinematic equation of AUV motion can be stated with the equation:

\[
x = cy\psi\theta u - (y\psi\phi + cy\theta\phi\psi\phi + s\psi\phi\phi + c\psi\phi\psi\phi)\]

\[
y = sy\psi\theta u + s\psi\phi\psi + s\phi\theta\psi\psi - c\psi\phi\phi + s\theta c\psi\phi\phi\]

(2.9)

(2.10)
\[ \dot{\psi} = c\phi q - s\phi r \]  

(2.11)

So surge, sway, and yaw can be stated in equation (2.15).

\[
\begin{bmatrix}
    m - Y_0 & mx_G - Y_0 & 0 \\
    -mx_G - N_0 & l_{xx} - N_r & 0 \\
    0 & 0 & 1
\end{bmatrix} \begin{bmatrix} \dot{v} \\ \dot{r} \\ \dot{\psi} \end{bmatrix} = [A] \begin{bmatrix} v \\ r \\ \psi \end{bmatrix} + [B] \begin{bmatrix} \delta_r \end{bmatrix} 
\]

(2.12)

Enter the parameter values from the plant shown in Table 2.1. Table 2.1 AUV Plant Parameters

[Figure 1 about here.]

Equation (2.15) results in linear and angular acceleration of AUV \((u, v, r)\), therefore, the acceleration data, can then be used to get the velocity of the AUV \((u, v, r)\). AUV speed data can be obtained by integrating the equation (2.15) (if the real plant data is taken from the estimation of the IMU sensor value) and position data from AUV relative to the earth's axis \((x, y, \psi)\) axis can be obtained by integrating it.

Figure 2. SDRE-LQT Tracking Control

When AUV moves in the horizontal plane, changes in rudder angle will produce a yaw moment and result in a change of direction towards AUV. In steering control, three states are required, namely, sway velocity \((v(t))\), yaw angle rate \((r(t))\), and yaw angle \((\psi(t))\). The mathematical equation of steering motion is:

\[ \dot{\psi} = \frac{\sin \phi}{\cos \theta} q + \frac{\cos \phi}{\cos \theta} r \]  

(2.19)

\[ m\dot{v} - mz_G\dot{\theta} + mx_G\dot{r} - Y_0\dot{v} - Y_0\dot{r} = mw_p - mur + my_G\dot{r}^2 + my_G^2\dot{\theta} - mz_Gq\dot{r} + Y_0v + Y_0v_0||v|| + Y_0u_r + Y_0wp + Y_0pwp + Y_0wpq + Y_0uuv + V_0u \theta \]  

(2.20)

\[ l_{xx}\dot{r} - mx_G\dot{v} + my_G\dot{u} - N_0\dot{v} - N_0\dot{r} = I_vy\dot{p} + I_{xx}\dot{p} - mx_Gwp + mx_Gur + my_G\dot{r} - my_Gwq + N_HS + N_u\dot{w} + N_uwp + N_uwpq + N_uu + N_uu_u^2 \delta_r \]  

(2.21)

If written in matrix form, the equations (2.24) - (2.26) are:

\[
A = \begin{bmatrix} Y_uu + Y_u v ||v|| & -mu + my_G r - mz_G q + Y_u u & 0 \\
my_G r + N_u u & mx_G u + N_u u & 0 \\
0 & \cos \phi & \cos \theta \end{bmatrix}
\]

(2.22)

\[
B = \begin{bmatrix} Y_{uu_u} \delta_r^2 \\
N_{uu_u} \delta_r \\
0 \end{bmatrix}
\]  

(2.23)
From equations (2.24) - (2.26) parameters can be sought for the state dependent coefficient by reducing all equations to each argument so that the parameterization obtained as in equation (2.27).

\[
\begin{bmatrix}
  m - Y_0 & m x_0 - Y_r & 0 \\
  -m x_0 - N_x & l_x - N_r & 0 \\
  0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
  \dot{\psi} \\
  \dot{\varphi} \\
  \dot{\delta}_v
\end{bmatrix} =
\begin{bmatrix}
  -\frac{\partial v}{u} & \frac{\partial v}{r} & \frac{\partial v}{\delta_v} \\
  \frac{\partial \varphi}{u} & \frac{\partial \varphi}{r} & \frac{\partial \varphi}{\delta_v} \\
  \frac{\partial \delta_v}{u} & \frac{\partial \delta_v}{r} & \frac{\partial \delta_v}{\delta_v}
\end{bmatrix} + \begin{bmatrix}
  m - Y_0 & m x_0 - Y_r & 0 \\
  -m x_0 - N_x & l_x - N_r & 0 \\
  0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
  v \\
  \varphi \\
  \delta_v
\end{bmatrix} 
\]

(2.27)

The design of SDRE-LQI control is based on the structure of servo 1 system, from the AUV dynamic equation (2.30) above, the linearization process using Jacobi matrix in equation (2.27) in the design of SDRE-LQI control uses servo 1 system, then state augmented must be made, so that figure 2 in closed loop form can be stated in the form of state augmented equation

\[
\begin{bmatrix}
  \dot{\bar{\chi}}_s (\infty) \\
  \dot{\bar{\xi}} (\infty) \\
  \dot{\bar{B}} u_{cc}(\infty) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} r(\infty)
\end{bmatrix} =
\begin{bmatrix}
  A & 0 \\
  -C & 0 \\
  0 & 1
\end{bmatrix} \begin{bmatrix}
  \bar{\chi}_s (\infty) \\
  \bar{\xi} (\infty) \\
  0
\end{bmatrix} + \begin{bmatrix}
  m - Y_0 & m x_0 - Y_r & 0 \\
  -m x_0 - N_x & l_x - N_r & 0 \\
  0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
  v \\
  \varphi \\
  \delta_v
\end{bmatrix} 
\]

(2.24)

(2.25)

So that equation (2.28) can be written in the state dependent coefficient (SDC) parameterization equation as in equation (2.29).

\[
\bar{\chi}_s = \bar{A}(\bar{\chi}_s) \bar{\chi}_s + \bar{B}(\bar{\chi}_s) u_{cc}
\]

(2.26)

Where, \(\bar{A}(\bar{\chi}_s) = \begin{bmatrix}
  Y_{uu} u + 2 Y_r |v| v \\
  -m u + 2 m y_{g} r - m z_0 q + Y_{vr} u \\
  m y_{gr} r + N_{uv} u \\
  m x_0 u + m y_{gv} + N_{ur}
\end{bmatrix}
\]

(2.28)

After we get the \(\bar{A}(\bar{\chi}_s)\) and \(\bar{B}(\bar{\chi}_s)\) matrix values, the next step is to determine the appropriate \(Q\) and \(R\) weighting matrices in order to get the correct gain feedback and control signal. The selection of \(Q\) and \(R\) matrices serves to minimize the Performance Index, as in Equation (2.32).

\[
\bar{\chi}_s = \bar{A}(\bar{\chi}_s) \bar{\chi}_s + \bar{B}(\bar{\chi}_s) u_{cc}
\]

(2.29)

Then with the matrix \(Q\) and \(R\), the next step is to solve the SDRE equation shown in Equation (2.33).

\[
\bar{A}^T(\bar{\chi}_s) P(\bar{\chi}_s) + P(\bar{\chi}_s) \bar{A}(\bar{\chi}_s) - P(\bar{\chi}_s) \bar{B}(\bar{\chi}_s) R^{-1}(\bar{\chi}_s) \bar{B}^T(\bar{\chi}_s) P(\bar{\chi}_s) + Q(\bar{\chi}_s) = 0
\]

(2.30)

This solution is used to obtain the Riccati matrix \(P(\bar{\chi}_s)\) with the help of matrices \(\bar{A}(\bar{\chi}_s), \bar{B}(\bar{\chi}_s)\), and the weighting matrix \(Q\) and \(R\), so that the calculation of Gain feedback and control signals in Equations (2.34) and (2.35) can be done.

\[
K = R^{-1}(\bar{\chi}_s) \bar{B}^T(\bar{\chi}_s) P(\bar{\chi}_s)
\]

(2.31)

\[
u_{cc} = -k_s x_s + k_\xi
\]

(2.32)

In this study, for the selection of weighting matrices, it is carried out through a trial and error process by considering the existing guidelines and choosing a matrix of \(Q\) and \(R\) values of:

\[
Q = \begin{bmatrix}
  27 & 0 & 0 \\
  0 & 27 & 0 \\
  0 & 0 & 29.5
\end{bmatrix}
\]

(2.33)

\[
R = 29.5
\]

(2.34)
III. RESULTS

This chapter discusses the results obtained from several different tests including the step system response. The purpose of testing the step response is to determine the system quality measurement. Testing is done by providing a reference signal in the form of a step signal. The step reference signal in this study uses the value of rudder deflection of 0.78 rad.

**Respons Step of Systems with SDRE-LQT**

The purpose of testing the step response is to determine the system quality measurement. Testing is done by providing a reference signal in the form of a step signal. The step reference signal given in this study uses the value of the maximum rudder deflection of 0.78 rad.

![Figure 3 about here.]

The step response angle yaw is shown in Figure 3. This is indicated by a time constant value of $\tau = 1.9904$ seconds. System response is also not too late to the input, there is only a delay of about $t_d = 1.3793$ seconds. The system response has appeared intact in an interval of about $t_r = 5.860$ seconds. It only takes a little time for the response to be around the steady state value, $t_s = 5.9703$ seconds. However, there is overshoot in transient conditions, with a maximum overshoot value of $M_p = 9.35\%$. In addition, the designed control system is also able to bring the system output according to the reference, which is 0.78 rad with a steady state error value $e = -0.01282\%$.

![Figure 4 about here.]

Figure 4 shows the step angle response of the pitch. This response arises by testing the forward movement of the AUV and the directed yaw angle of 0.78 rad. It can be chosen that $t_p = 1.914$ seconds and $M_p = 27.97\%$. In addition, the control system designed is also able to bring the pitch angle according to the reference, which is 0 radians. In addition, the control system designed is also able to bring the pitch angle according to the reference, which is 0 radians.

![Figure 5 about here.]

Whereas the step response for roll angles is shown in Figure 5. It was clearly seen that the control system that was designed was able to stabilize the roll angle, namely by showing the response of the roll angle to be around 0 radians. Roll angle deviation that occurs at the beginning, due to the effect of changing the yaw and pitch angle when the AUV starts moving forward.

IV. CONCLUSION

The SDRE-LQT control method works quite well when there is a nonlinearity factor of system, namely the influence of roll angle and pitch angle that affects yaw angle state, which causes overshoot and undershoot, where SDRE-LQT controller is able to control the AUV yaw angle according to change reference signal given with a small steady state error, $e = -0.01282\%$.

V. SUGGESTION

Suggestions for further research can discuss yaw angle orientation control by considering the addition of sea current disturbance which causes the parameters to be uncertainty.

DAFTAR PUSTAKA


Conflict of Interest Statement: The author declares that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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Received: 2019-09-06
Accepted: 2019-10-25
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